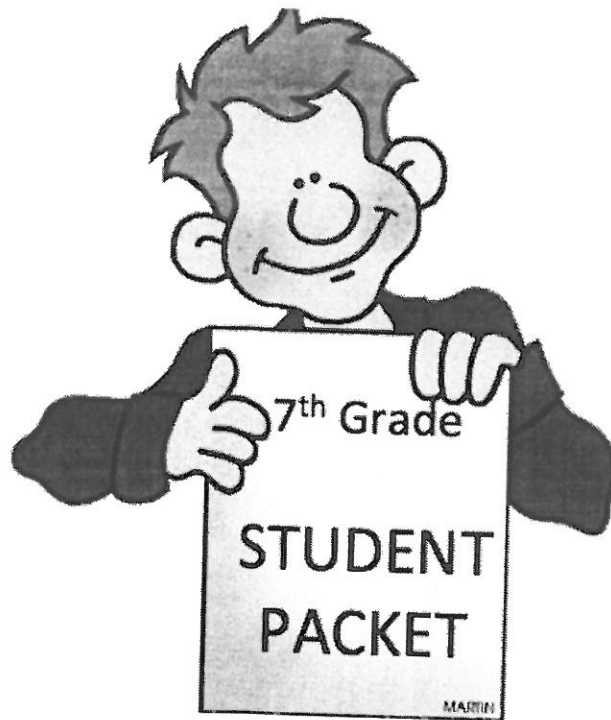


7th Grade

Intensive Math



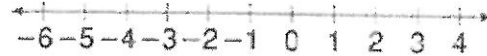
LESSON
2-3

Practice A
Subtracting Integers

Show the subtraction on the number line. Then write the difference.

1. $3 - 8$

2. $-5 - (-1)$



Find each difference.

3. $-3 - 4$

4. $-7 - (-2)$

5. $12 - 6$

6. $2 - (-7)$

7. $-8 - 8$

8. $-5 - (-5)$

9. $-1 - (-2)$

10. $9 - (-3)$

11. $8 - 1$

12. $7 - (-9)$

13. $-3 - 8$

14. $-3 - (-7)$

Evaluate $x - y$ for each set of values.

15. $x = 6, y = -3$

16. $x = -7, y = 1$

17. $x = -2, y = -5$

18. $x = 9, y = 11$

19. $x = -1, y = -1$

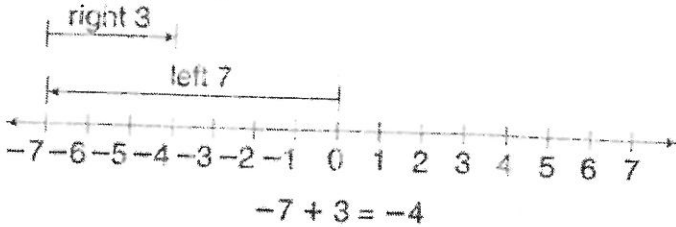
20. $x = -5, y = 5$

21. The high temperature one day was 6°F . The low temperature was -3°F . What was the difference between the high and low temperatures for the day?

22. The temperature changed from -7°F at 6 A.M. to 7°F at noon. How much did the temperature increase?

LESSON **2-3** **Reading Strategies**
Use Graphic Aids

Brett borrowed \$7 from his father to buy a CD. He paid back \$3. How much money does Brett have now? The number line will help you picture this problem.

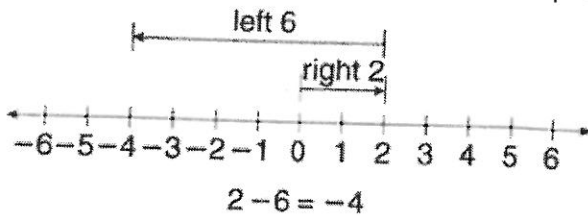


1. Beginning at 0, in which direction will you move first? _____
2. How many places? _____
3. Which direction do you move next? _____
4. How many places? _____
5. On what number do you end? _____

Bret does not have any money. He owes his dad \$4. He has negative \$4.

Sally and her friends made up a game with points. You can either win or lose up to ten points on each round of the game. After the first round, Sally's team had 2 points. In the second round they lost 6 points. How many points was Sally's team down by after the second round?

The number line will help you picture the problem.



6. Beginning at zero, which direction will you move first? How many places?

7. Which direction will you move next? How many places?

8. By how many points was Sally's team down? _____

LESSON
2-3 **Review for Mastery**
Subtracting Integers

The total value of the three cards shown is -6 .

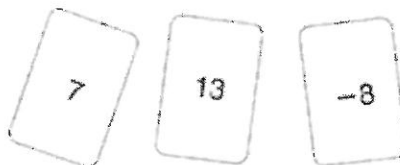


What if you **take away** the 3 card?
Cards -4 and -5 are left.
The new value is -9 .
 $-6 - 3 = -9$

What if you **take away** the -4 card?
Cards 3 and -5 are left.
The new value is -2 .
 $-6 - (-4) = -2$

Answer each question.

1. Suppose you have the cards shown.
The total value of the cards is 12.



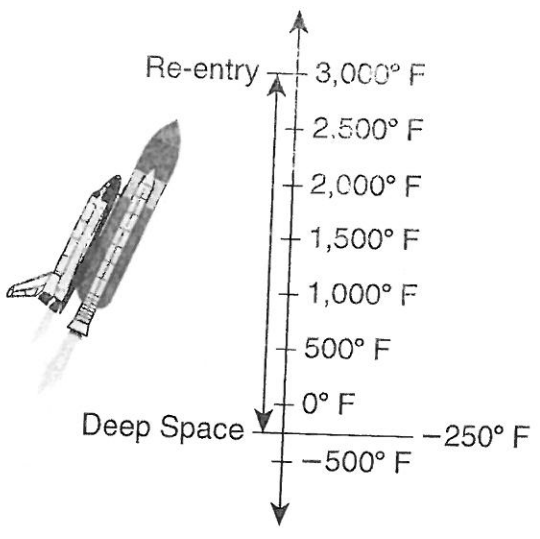
- a. What if you take away the 7 card? $12 - 7 =$ _____
- b. What if you take away the 13 card? $12 - 13 =$ _____
- c. What if you take away the -8 card? $12 - (-8) =$ _____
2. Subtract $-4 - (-2)$.
- a. $-4 < -2$. Will the answer be positive or negative? _____
- b. $|4| - |2| =$ _____
- c. $-4 - (-2) =$ _____
3. Subtract $21 - 13$.
- a. $21 > 13$. Will the answer be positive or negative? _____
- b. $|21| - |13| =$ _____
- c. $21 - 13 =$ _____

Subtract.

4. $31 - (-9) =$ _____
5. $15 - 18 =$ _____
6. $-9 - 17 =$ _____
7. $-8 - (-8) =$ _____
8. $29 - (-2) =$ _____
9. $13 - 18 =$ _____

LESSON 2-3 Student Worksheet
Subtracting Integers

Problem 1



Problem 2

-
 means to
ADD
 the opposite.

$$5 - 9 = 5 - (+9)$$

$$= 5 + (-9)$$

$$= -4$$

The opposite of
 + is -.
 The opposite of
 - is +.

$$-4 - 3 = -4 + (-3)$$

$$= -7$$

Think and Discuss

1. Why do you add 3,000° and 250° in Problem 1?

2. In Problem 2, what is the opposite of 9? _____
3. Why do you not change the -4 to +4 in Problem 2?

4. Is 3 - 5 the same as 5 - 3? Explain.

LESSON
2-4**Practice A****Multiplying and Dividing Integers**

Find each product.

1. $6 \cdot (-1)$

2. $-4 \cdot 2$

3. $-3 \cdot (-4)$

4. $-2 \cdot 8$

5. $5 \cdot (-7)$

6. $-7 \cdot 9$

7. $8 \cdot 4$

8. $-3 \cdot (-5)$

9. $-5 \cdot (-5)$

10. $8 \cdot (-4)$

11. $-7 \cdot (-6)$

12. $9 \cdot (-8)$

13. $1 \cdot (-7)$

14. $-4 \cdot (-5)$

15. $-6 \cdot 3$

16. $-7 \cdot (-7)$

Find each quotient.

17. $12 \div (-4)$

18. $-15 \div (-3)$

19. $-20 \div 5$

20. $-27 \div (-9)$

21. $-45 \div (-5)$

22. $-18 \div 9$

23. $24 \div (-4)$

24. $32 \div 4$

25. $21 \div 3$

26. $-36 \div (-4)$

27. $16 \div (-4)$

28. $-56 \div 8$

29. $-42 \div 7$

30. $-30 \div (-6)$

31. $27 \div 9$

32. $25 \div 0$

33. A scientist is measuring the temperature change in a chemical compound. The temperature dropped 11°F per hour from the original temperature. After 4 hours, the temperature was 90°F . Find the compound's original temperature.
- _____

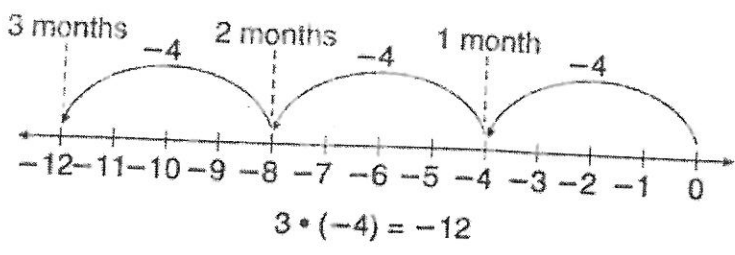
34. A mountain climber ascends 800 feet per hour from his original position. After 6 hours, his final position is 11,600 feet above sea level. Find the climber's original position.
- _____

LESSON 2.4 **Reading Strategies**
Use Graphic Aids

The opposite of 5 is negative 5. Owing money is the opposite of having money. Owing \$5 is the opposite of having \$5.

1. What is the opposite of owing \$10? _____
2. What is the opposite of having \$17? _____

David borrowed \$4 from his mother each of the last three months. How much money does he owe his mother? The money he owes his mother is a negative number. This problem can be pictured on a number line.



Use the number line to help you answer the questions.

3. Starting at zero, which direction do you move first? _____
4. How many places do you move? _____
5. Which direction do you move next? _____
6. How many places do you move? _____
7. Which direction do you move next? _____
8. How many places do you move? _____
9. How much money does David owe his mother? _____
10. If David borrowed \$4 for one more month, how much would he owe his mother? _____

LESSON
2-4

Review for Mastery
Multiplying and Dividing Integers

Look for the patterns in these products and quotients.

$1 \cdot 3 = 3$	$-1 \cdot 3 = -3$	$3 \div 1 = 3$	$3 \div (-1) = -3$
$2 \cdot 3 = 6$	$-2 \cdot 3 = -6$	$6 \div 2 = 3$	$6 \div (-2) = -3$
$-3 \cdot (-3) = 9$	$3 \cdot (-3) = -9$	$-9 \div (-3) = 3$	$-9 \div 3 = -3$
$-4 \cdot (-3) = 12$	$4 \cdot (-3) = -12$	$-12 \div (-4) = 3$	$-12 \div 4 = -3$

Look at how to find the signs of the products.

- The product of two integers with the **same sign** is **positive**.

$$(+)\cdot(+)=(+)$$

$$(-)\cdot(-)=(+)$$

- The product of two integers with **different signs** is **negative**.

$$(+)\cdot(-)=(-)$$

$$(-)\cdot(+)=(-)$$

Look at how to find the signs of the quotients.

- The quotient of two integers with the **same sign** is **positive**.

$$(\div)(\div)=(+)$$

$$(-)\div(-)=(+)$$

- The quotient of two integers with **different signs** is **negative**.

$$(\div)(-)=(-)$$

$$(-)\div(+)=(-)$$

Find each product or quotient.

1. $-5 \cdot 4$

2. $2 \cdot (-8)$

3. $-1 \cdot (-1)$

4. $-6 \cdot 3$

5. $7 \cdot (-3)$

6. $-8 \cdot (-4)$

7. $-6 \cdot 5$

8. $-9 \cdot (-9)$

9. $36 \div (-4)$

10. $-27 \div 9$

11. $-24 \div (-6)$

12. $-30 \div 5$

13. $18 \div 6$

14. $32 \div (-8)$

15. $-45 \div 9$

16. $-40 \div (-10)$

LESSON 2.4 **Student Worksheet**
Multiplying and Dividing Integers

Problem 1

The rules for multiplying and dividing integers are the same.

Same signs \longrightarrow Positive

$$\begin{array}{ll} (+) \cdot (+) = + & (-) \cdot (-) = + \\ (+) \div (+) = + & (-) \div (-) = + \end{array}$$

Different signs \longrightarrow Negative

$$\begin{array}{ll} (-) \cdot (+) = - & (+) \cdot (-) = - \\ (+) \div (-) = - & (-) \div (+) = - \end{array}$$

Determine if each product or quotient is positive, +, or negative, -.

$(-3) \cdot (-3) \longrightarrow$ positive, +

$6 \div (-3) \longrightarrow$ negative, -

Problem 2

When dividing integers, follow these steps:
 1. Divide the integers.
 2. Look at the signs of each number to give the answer a sign.

$-100 \div (-5)$
Think: $- \div - = +$
 $-100 \div (-5) = +20$

Think and Discuss

1. Why is the quotient of $-100 \div (-5)$ the same as the quotient of $100 \div 5$?

2. Is $6 \div (-3)$ the same as $-6 \div 3$? Explain.

LESSON
2-11

Practice A
Equivalent Fractions and Decimals

Write each fraction as a decimal. Round to the nearest hundredth, if necessary.

1. $\frac{2}{3}$ _____

2. $\frac{9}{20}$ _____

3. $\frac{3}{4}$ _____

4. $\frac{20}{25}$ _____

5. $\frac{3}{8}$ _____

6. $\frac{7}{5}$ _____

7. $\frac{21}{7}$ _____

8. $\frac{5}{3}$ _____

9. $\frac{4}{9}$ _____

10. $\frac{4}{5}$ _____

11. $\frac{1}{25}$ _____

12. $\frac{3}{20}$ _____

Write each decimal as a fraction or mixed number in simplest form.

13. 0.55

14. 0.03

15. -0.75

16. 2.1

17. 5.25

18. 9.33

19. 1.8

20. -1.74

21. 10.6

22. -7.08

23. 0.625

24. 0.001

Write each answer as a decimal rounded to the nearest thousandth.

25. Out of 45 times at bat, Raul got 19 hits. Find Raul's batting average.

26. On a test, Selena answered 26 out of 30 questions correctly. What portion of her answers was correct?

LESSON **2-11** **Reading Strategies**

Compare and Contrast

Compare what happens when fractions are changed to decimals.

$\frac{2}{5}$ • Read $\frac{2}{5}$ as "2 divided by 5." • Write $\rightarrow 2 \div 5$

Change a fraction to a decimal by dividing the numerator by the denominator.

$$\begin{array}{r} 0.4 \\ 5 \overline{)2.0} \end{array}$$

$\frac{2}{5} = 0.4$ The dividing ends, or terminates, with no remainder. 0.4 is called a terminating decimal.

1. Is there a remainder in the problem? How do you know?

2. What do we call a decimal that ends with no remainder?

$\frac{2}{6}$ • Read $\frac{2}{6}$ as "2 divided by 6." • Write $\rightarrow 2 \div 6$

$$\begin{array}{r} 0.333 \\ 6 \overline{)2.000} \end{array}$$

$$\begin{array}{r} -18 \\ 20 \end{array}$$

$$\begin{array}{r} -18 \\ 20 \end{array}$$

$\frac{2}{6} = 0.333 \dots$ or $0.\overline{3}$

Note how dividing continues in a pattern. The number 0.333... is a repeating decimal. The bar over the 3 means 3 repeats.

Answer each question.

3. Compare the division of $\frac{2}{5}$ to the division of $\frac{2}{6}$. What is the difference?

4. What is the name for a decimal with a remainder that has a repeating pattern?

LESSON
2-11

Review for Mastery
Equivalent Fractions and Decimals

To write a fraction as a decimal, divide the numerator of the fraction by the denominator of the fraction.

Write $\frac{3}{7}$ as a decimal.

- Divide 3 by 7.
- To round your answer to the nearest hundredth, add 3 zeros after the decimal point in the divisor.

0.428 rounded to the nearest hundredth is 0.43.

$$\begin{array}{r} 0.428 \\ 7 \overline{)3.000} \\ \underline{-28} \\ 20 \\ \underline{-14} \\ 60 \\ \underline{-56} \\ 4 \end{array}$$

1. Write $\frac{2}{5}$ as a decimal.

$$\frac{2}{5} = \underline{\hspace{2cm}}$$

$$\begin{array}{r} 5 \overline{)2.0} \\ \underline{-} \\ \underline{\hspace{1cm}} \end{array}$$

Write each fraction as a decimal. Round to the nearest thousandth, if necessary.

2. $\frac{3}{4}$ _____

3. $\frac{7}{8}$ _____

4. $\frac{3}{2}$ _____

5. $\frac{5}{3}$ _____

To write a decimal as a fraction:

Step 1: Use place value to read the decimal. Say the number aloud.

Step 2: Write a fraction for the number you just said.

Step 3: Simplify if necessary.

Write 0.005 as a fraction.

Read 0.005 as "five thousandths."

Write $\frac{5}{1000}$ for five thousandths.

Simplify: $\frac{5 \div 5}{1,000 \div 5} = \frac{1}{200}$

Write 1.6 as a fraction.

Read 1.6 as "one and six tenths."

Write $1\frac{6}{10}$ for one and six tenths.

Simplify: $1\frac{6 \div 2}{10 \div 2} = 1\frac{3}{5}$

Write each decimal as a fraction or mixed number in simplest form.

6. 0.8 _____

7. 2.25 _____

8. -0.02 _____

LESSON 2-11 **Student Worksheet**
Equivalent Fractions and Decimals

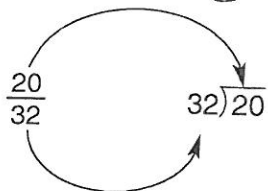
Problem 1



So far, I have 20 hits out of 32 at bats. What is my average?

$$\begin{aligned} \text{Average} &= \frac{\text{hits}}{\text{at bats}} \\ &= \frac{20}{32} \end{aligned}$$

$\frac{20}{32} =$ What decimal?



$$\begin{array}{r} \frac{20}{32} = 32 \overline{)20} = 32 \overline{)20.000} \\ \underline{-192} \\ 80 \\ \underline{-64} \\ 160 \\ \underline{-160} \\ 0 \end{array}$$

His batting average is 0.625.

Problem 2



What is 0.036?

6 is in the "thousandths" position on the place value chart.

$$\begin{array}{r} 0.036 \longrightarrow \frac{36}{1,000} \\ \frac{36}{1,000} \div 4 = \frac{9}{250} \end{array}$$

Think and Discuss

1. Is the baseball average in Problem 1 a terminating or repeating decimal? Explain.

2. What is the place value of the 6 in 0.625? _____
3. Complete: 0.036 = thirty-six- _____
4. Are these two decimals different? Explain.

$0.3333333333333333\dots$ 0.3

LESSON
3-4 **Practice A**
Multiplying Decimals

Multiply. Choose the letter for the best answer.

- | | | | |
|--------------------|---------|------------------|---------|
| 1. $5 \cdot 0.05$ | | 2. $9 \cdot 0.7$ | |
| A 25 | C 0.25 | F 63 | H 0.63 |
| B 2.5 | D 0.025 | G 6.3 | I 0.063 |
| 3. $6 \cdot 0.003$ | | 4. $5 \cdot 1.2$ | |
| A 18 | C 0.18 | F 60 | H 0.6 |
| B 1.8 | D 0.018 | G 6 | I 0.06 |

Simplify. Choose the letter for the best answer.

- | | | | |
|------------------|----------|-------------------|---------|
| 5. $6 \cdot 1.8$ | | 6. $(0.4)^2$ | |
| A 10.8 | C 0.108 | F 16 | H 0.16 |
| B 1.08 | D 0.0108 | G 1.6 | I 0.016 |
| 7. $3 \cdot 8.4$ | | 8. $7 \cdot 0.51$ | |
| A 25.2 | C 0.252 | F 357 | H 3.57 |
| B 2.52 | D 0.0252 | G 35.7 | I 0.357 |

Multiply. Estimate to check whether each answer is reasonable.

- | | | |
|-----------------------|----------------------|----------------------|
| 9. $6.8 \cdot 4$ | 10. $8.1 \cdot (-2)$ | 11. $9.5 \cdot 5$ |
| _____ | _____ | _____ |
| 12. $3.5 \cdot 7$ | 13. $-6.3 \cdot 6$ | 14. $9 \cdot 3.7$ |
| _____ | _____ | _____ |
| 15. $-6.7 \cdot (-5)$ | 16. $8.8 \cdot (-8)$ | 17. $5.2 \cdot (-4)$ |
| _____ | _____ | _____ |
| 18. $-3 \cdot 4.1$ | 19. $1.5 \cdot 1.2$ | 20. $-2.3 \cdot 1.7$ |
| _____ | _____ | _____ |
21. Cecile walked 3.7 miles each day for 8 days last month.
How many miles total did Cecile walk last month?
- _____

LESSON **3-4** **Reading Strategies**

Compare and Contrast

Decimals are multiplied in much the same way that you multiply whole numbers.

Multiply Whole Numbers

$$\begin{array}{r} 5 \\ \times 7 \\ \hline 35 \end{array}$$

Multiply Decimals

$$\begin{array}{r} 0.5 \\ \times 0.7 \\ \hline 0.35 \end{array}$$

Compare multiplying whole numbers to multiplying decimals.

1. What is the same about multiplying whole numbers and decimals?

2. What is different about multiplying whole numbers and decimals?

It is important to place the decimal point correctly in the product.

Steps for Placing the Decimal Point in the Product	Example: 1.37×0.8
Step 1: Find the product.	1096
Step 2: Count the number of decimal places in each factor.	$1.37 \rightarrow 2$ places $0.8 \rightarrow 1$ place
Step 3: Find the total number of decimal places in both numbers.	3 places
Step 4: Using the number found in Step 3, move that number of places to the left in the product and place the decimal point.	1.096

3. How many decimal places are in 0.63?

4. How many decimal places are in 4.231?

5. How many decimal places will be in the product of 0.63×4.231 ?

LESSON
3-4

Review for Mastery

Multiplying Decimals

To multiply two decimals:

- Step 1:** Round each number to the nearest integer.
- Step 2:** Multiply the integers to estimate the product.
- Step 3:** Multiply the decimals.
- Step 4:** Place the decimal point in the product to make it closest to the estimate.

Multiply: $2.7 \cdot 4.3$

$$\begin{array}{r} 4.3 \\ 2.7 \\ \hline 301 \\ 860 \\ \hline 11.61 \end{array}$$

11.61 is close to 12.

Think:

2.7 rounds to 3.
4.3 rounds to 4.
 $3 \cdot 4 = 12$
Place the decimal point in the product to make it closest to 12.

Multiply or simplify.

1. $6.7 \cdot 9.1$

6.7 rounds to _____

9.1 rounds to _____

The product is close to _____.

Product: _____

3. $(4.1)^2$

4.1 rounds to _____

The product is close to _____.

Product: _____

2. $-3.21 \cdot 8.8$

-3.21 rounds to _____

8.8 rounds to _____

The product is close to _____.

Product: _____

4. $12.3 \cdot (-2.7)$

12.3 rounds to _____

-2.7 rounds to _____

The product is close to _____.

Product: _____

Simplify. Estimate to place the decimal point.

5. $2.06 \cdot 7.9$

7. $8.23 \cdot (-4.2)$

6. $-4.89 \cdot 0.6$

8. $(5.3)^2$

LESSON **Student Worksheet**
3-4 **Multiplying Decimals**

Problem 1

Multiply.

$1.25 \cdot 23$

$1.25 \Rightarrow 2$ decimal places
 1 2

$23 \Rightarrow 0$ decimal places

$2 + 0 = 2$ decimal places \Rightarrow
 in the answer

$$\begin{array}{r} 1.25 \\ \times 23 \\ \hline 375 \\ + 2500 \\ \hline 28.75 \end{array}$$

Problem 2

Multiply.

$1.2 \cdot 1.6$

$1.2 \Rightarrow 1$ decimal place
 1

$1.6 \Rightarrow 1$ decimal place
 1

$1 + 1 = 2$ decimal places \Rightarrow
 in the answer

$$\begin{array}{r} 1.2 \\ \times 1.6 \\ \hline 72 \\ + 120 \\ \hline 1.92 \end{array}$$

Think and Discuss

1. Explain how to determine the number of decimal places in the product of a multiplication problem involving decimal factors.

2. To place the decimal point in the product of two decimals, do you move the decimal point to the left or do you move the decimal point to the right?

3. Explain how to determine if your answer to Problem 1 is reasonable.

LESSON
3-5**Practice A**
Dividing Decimals**Divide. Estimate to check whether your answer is reasonable.**

1. $7.5 \overline{)15}$

2. $1.2 \overline{)72}$

3. $1.5 \overline{)45}$

4. $7.5 \overline{)-22.5}$

5. $4.8 \overline{)16.8}$

6. $-2.7 \overline{)11.07}$

Divide. Estimate to check whether your answer is reasonable.

7. $2.8 \overline{)14}$

8. $-5.6 \overline{)21}$

9. $3.2 \overline{)48}$

10. $2.25 \overline{)9}$

11. $2.4 \overline{)6}$

12. $-1.25 \overline{)65}$

13. Jessie used 2.7 gallons of gas to drive her car 72.9 miles. What was her car's gas mileage?

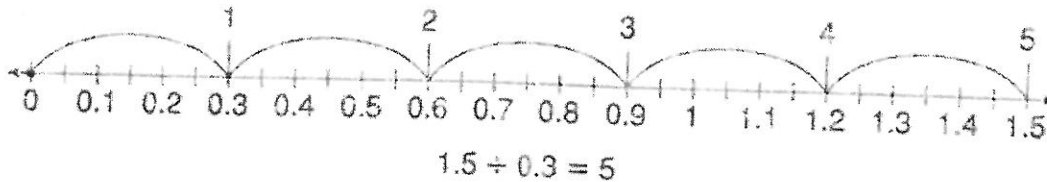
14. Ernesto bicycled 267 miles last week at an average speed of 8.9 mi/h. How many hours did he bicycle?

LESSON
3-5

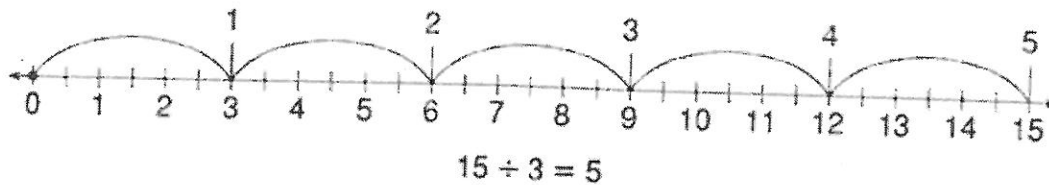
Reading Strategies

Use a Visual Model

John has a piece of lumber 1.5 meters long. He needs to cut it into pieces that are 0.3 meter long. How many pieces can he cut? The number line shows a model of the problem.



Sarah has 15 feet of yarn. She needs to cut it into lengths of 3 feet each. How many pieces can she cut? The number line shows a model of the problem.



Answer each question.

1. Compare the equations for the number lines above. What is the same about the equations?

2. What is different?

3. Compare the quotients of both problems. What do you notice?

4. How can you change 1.5 to 15?

5. How can you change 0.3 to 3?

6. If you moved the decimal point in **both** the divisor and the dividend, would the quotient change?

LESSON
3-5

Review for Mastery

Dividing Decimals

To divide a decimal by a decimal:

Step 1: Make the divisor a whole number by moving the decimal point to the right.

Step 2: Move the decimal point in the dividend the same number of places. Remember to place the decimal in the quotient directly above the decimal point in the dividend.

Step 3: Divide.

Divide: $1.68 \div 0.3 \rightarrow 0.3 \overline{)1.68} \rightarrow 3 \overline{)16.8}$

$$\begin{array}{r} 5.6 \\ 3 \overline{)16.8} \\ \underline{-15} \downarrow \\ 18 \\ \underline{-18} \end{array}$$

The divisor, 0.3, has 1 decimal place. Move the decimal point 1 place to the right in both the divisor and the dividend.

Complete.

1. $5.6 \overline{)4.48}$

- a. How many decimal places are in the divisor? _____
- b. How many places do you need to move each decimal point? _____
- c. Rewrite the division. _____
- d. Complete the division. What is the quotient? _____

Divide.

2. $5.2 \overline{)3.64}$

3. $0.09 \overline{)36.45}$

4. $0.59 \overline{)0.708}$

LESSON
3-5

Review for Mastery

Dividing Decimals (continued)

Sometimes it is necessary to write zeros in the dividend.

$$6 \div 0.25 \rightarrow 0.25 \overline{)6} \rightarrow 0.25 \overline{)6.00} \rightarrow 25 \overline{)600}$$

$$\begin{array}{r} 24 \\ 25 \overline{)600} \\ \underline{-50} \downarrow \\ 100 \\ \underline{-100} \end{array}$$

The divisor, 0.25, has 2 decimal places. In order to move the decimal point 2 places to the right in the divisor and the dividend, you need to write 2 zeros in the dividend.

Complete.

5. $0.35 \overline{)7}$

- a. How many decimal places are in the divisor? _____
- b. How many places do you need to move each decimal point? _____
- c. How many zeros do you need to write in the dividend? _____
- d. Complete the division. What is the quotient? _____

Divide.

6. $1.6 \overline{)8}$

7. $0.12 \overline{)19.2}$

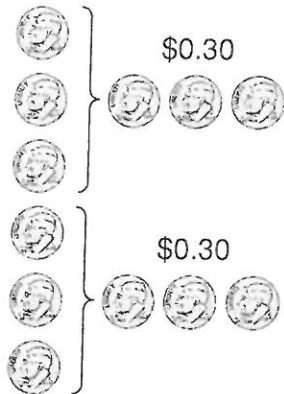
8. $1.25 \overline{)48}$

LESSON
3-5 **Student Worksheet**
Dividing Decimals

Problem 1

How many groups of \$0.30 are there?

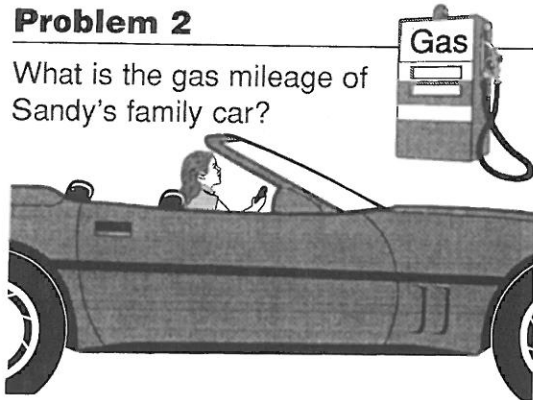
\$0.60



There are 2 groups. $\$0.60 \div \$0.30 = 2$

Problem 2

What is the gas mileage of Sandy's family car?



Sandy drove 358.8 miles.
 The car used 14.95 gallons of gas.

Gas Mileage:

$$\frac{\text{miles driven} \rightarrow 358.8 \text{ miles}}{\text{gallons used} \rightarrow 14.95 \text{ gallons}}$$

$$14.95 \overline{) 358.80} \quad \text{Divide}$$

Gas mileage: 24 miles per gallon

Think and Discuss

1. What do you multiply the divisor and the dividend by in Problem 2 to eliminate the decimal point in the divisor? _____
2. When dividing a decimal by a decimal, why can you move the decimal points?

Name _____

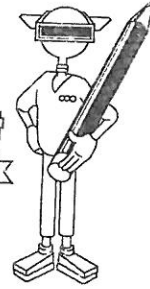
Date _____



COURSE: MSC IV
MODULE 2: Decimals
UNIT 4: Dividing Decimals

Estimating and Finding Quotients

Student
Logbook



As you work through the tutorial, complete the following statements and questions.

1. What do kilowatts and horsepower measure? _____
2. How much power in kilowatts is needed for the fountain lights and pump? _____
3. According to the *Earth Guide*, 1 horsepower is equal to _____ kilowatt.
4. What expression describes the power in horsepower that is needed for the fountain lights and pump? _____
5. Why does Dijit multiply $\frac{1.865}{0.746}$ by $\frac{1,000}{1,000}$? _____

6. Does multiplying by $\frac{1,000}{1,000}$ change the value of the fraction? _____
Explain your answer. _____

7. In order to estimate the horsepower needed for the fountain lights and pump, Dijit and Jack rounded each decimal number to the nearest _____. The estimated power needed is _____ kW.
8. Why does Dijit add a decimal point and a zero to the dividend? _____

9. To divide decimals, first multiply the divisor by a power of _____ to make it a _____ number. Then, multiply the dividend by the same _____ of _____ before you divide.

Key Words:

Decimal
Division

Learning Objectives:

- Expressing a decimal denominator as a whole number by multiplying the numerator and denominator of the fraction by a power of 10
- Dividing a decimal number by a decimal number
- Adding zeros to the right of a decimal point to act as place holders in a dividend
- Estimating an answer when dividing by decimals



COURSE: **MSC IV**
 MODULE 2: **Decimals**
 UNIT 4: **Dividing Decimals**

Estimating and Finding Quotients



1. **a.** In the problem $3.7 \overline{)108.41}$, what is the first step? _____

 - b.** Find the quotient of $6.3 \overline{)236.88}$ _____
 - c.** $1,584 \div 13.2 =$ _____
 - d.** $87.63 \div 6.35 =$ _____
2. A tire manufacturer uses the formula $C = \pi d$ to calculate the meter circumference of a tire, where d represents the diameter of the tire and $\pi = 3.14$.
 - a.** Estimate the diameter of the tire to the nearest whole number. _____
 - b.** Calculate the diameter of the tire to the nearest hundredth. _____
 - c.** Check your answer to part (b) by multiplying the divisor and the quotient. Show your work.
 3. The watt is a unit of power, and 1 kilowatt (kW) = 1,000 watts.
 - a.** After 9.5 hours, a meter reads 13.56 kilowatt-hours (kWh). How many kilowatt-hours were used during one hour? Round your answer to the nearest hundredth. _____
 - b.** If an electric bill shows a total of 2,977.2 kWh used at a rate of 4.135 kWh per hour, how long was the billing cycle? _____

LESSON
3-10
Practice A
Multiplying Fractions and Mixed Numbers

Simplify. Choose the letter for the best answer.

1. $\left(\frac{3}{8}\right)^2$

A $\frac{9}{64}$

B $\frac{6}{16}$

3. $4 \cdot 3\frac{3}{5}$

A $2\frac{2}{5}$

B 12

2. $\frac{2}{5} \cdot \frac{3}{4}$

F $\frac{1}{4}$

G $\frac{2}{3}$

H $\frac{3}{10}$

I $\frac{5}{9}$

4. $1\frac{1}{4} \cdot 2\frac{2}{3}$

F $2\frac{1}{6}$

G $3\frac{1}{3}$

H $3\frac{2}{3}$

I $3\frac{11}{12}$

Simplify. Write each answer in simplest form.

5. $\left(\frac{1}{3}\right)^2$

6. $8 \cdot \frac{1}{4}$

7. $10 \cdot \frac{1}{5}$

8. $\frac{1}{2} \cdot \frac{1}{4}$

9. $\frac{1}{4} \cdot \left(-\frac{2}{3}\right)$

10. $\left(\frac{2}{3}\right)^3$

11. $-16 \cdot \frac{3}{4}$

12. $24 \cdot \frac{5}{6}$

13. $32 \cdot \frac{3}{8}$

14. $2\frac{1}{4} \cdot \frac{1}{2}$

15. $3\frac{1}{3} \cdot \frac{3}{5}$

16. $5\frac{1}{3} \cdot \frac{1}{4}$

17. $1\frac{1}{2} \cdot 1\frac{1}{5}$

18. $1\frac{2}{5} \cdot 2\frac{3}{4}$

19. $2\frac{2}{7} \cdot 3\frac{1}{8}$

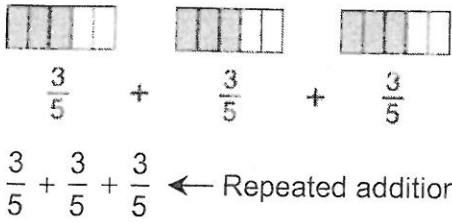
20. Louis spent 12 hours last week practicing guitar. If $\frac{1}{4}$ of the time was spent practicing chords, how much time did he spend practicing chords?
- _____

LESSON
3-10

Reading Strategies

Use Fraction Strips

You can write a multiplication problem as a repeated addition problem.



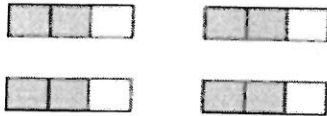
Use the fraction strips above to answer questions 1–4.

1. What fractional part of the fraction strips is shaded? _____
2. How many fraction strips are there? _____
3. Count the number of fractional parts that are shaded in all.
How many are there? _____
4. How can you find the answer to the problem above using addition?

You can also find the answer to the above problem using

multiplication. $3 \times \frac{3}{5} = \frac{9}{5}$

Use the fraction strips below to answer questions 5–7.



5. What fractional part of each fraction strip is shaded? _____
6. How many of these fraction strips are there? _____
7. Write a multiplication equation for this picture.

LESSON
3-10
Review for Mastery
Multiplying Fractions and Mixed Numbers

To multiply fractions and mixed numbers:

Step 1: Write any mixed numbers as improper fractions.

Step 2: Multiply the numerators.

Step 3: Multiply the denominators.

Step 4: Write the answer in simplest form.

Multiply: $\frac{4}{9} \cdot \frac{3}{8}$

$$\begin{aligned} \frac{4}{9} \cdot \frac{3}{8} &= \frac{4 \cdot 3}{9 \cdot 8} \\ &= \frac{12}{72} \\ &= \frac{1}{6} \end{aligned}$$

Divide numerator and denominator by 12, the GCF.

Multiply: $6\frac{1}{4} \cdot -1\frac{4}{5}$

$$\begin{aligned} 6\frac{1}{4} \cdot -1\frac{4}{5} &= \frac{25}{4} \cdot \frac{-9}{5} \\ &= \frac{25 \cdot (-9)}{4 \cdot 5} \\ &= \frac{-225}{20} \\ &= -11\frac{1}{4} \end{aligned}$$

Remember, positive times negative equals negative.

Multiply. Write each answer in simplest form.

1. $6 \cdot \frac{1}{9} = \frac{6 \cdot 1}{9} = \frac{\quad}{\quad}$

2. $-\frac{4}{5} \cdot \frac{5}{7} = -\frac{4 \cdot \quad}{5 \cdot \quad} = \frac{\quad}{\quad}$

3. $3\frac{1}{3} \cdot 9 = \frac{10}{3} \cdot 9 = \frac{10 \cdot \quad}{\quad} = \frac{\quad}{\quad}$

4. $\frac{3}{10} \cdot 2\frac{1}{2} = \frac{3}{10} \cdot \frac{5}{2} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

5. $\left(\frac{1}{2}\right)^2$

6. $-\frac{5}{9} \cdot \frac{3}{4}$

7. $\frac{9}{10} \cdot -\frac{2}{3}$

8. $2\frac{5}{8} \cdot \frac{2}{3}$

9. $\frac{1}{2} \cdot 4\frac{1}{4}$

10. $-\frac{2}{3} \cdot 1\frac{3}{4}$

11. $5\frac{1}{5} \cdot -1\frac{2}{3}$

12. $4\frac{1}{2} \cdot 1\frac{1}{9}$

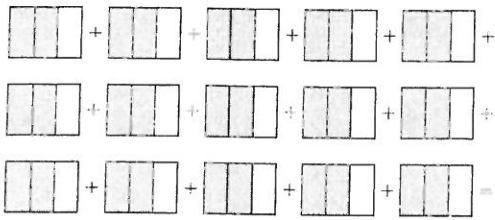
13. $-2\frac{3}{4} \cdot -1\frac{1}{3}$

LESSON
3-10 **Student Worksheet**
Multiplying Fractions and Mixed Numbers

Problem 1

Multiply. $-15 \cdot \frac{2}{3}$

$-15 \cdot \frac{2}{3}$ is the same as 15 groups of $\frac{2}{3}$.



$$\frac{30}{3} = 10$$

Since the signs are different, the product is -10 .

Problem 2

Multiply. $\frac{1}{3} \cdot 4\frac{1}{2}$

Write $4\frac{1}{2}$ as an improper fraction.

$$4\frac{1}{2} = \frac{4 \cdot 2 + 1}{2} = \frac{8 + 1}{2} = \frac{9}{2}$$

$$\frac{1}{3} \cdot \frac{9}{2} = \frac{9}{6} = \frac{3}{2}$$

Write $\frac{3}{2}$ as a mixed number.

$$\begin{array}{r} 1 \\ 2 \overline{)3} = 1\frac{1}{2} \\ -2 \\ \hline 1 \end{array}$$

$$\frac{1}{3} \cdot 4\frac{1}{2} = \frac{1}{3} \cdot \frac{9}{2} = \frac{3}{2} = 1\frac{1}{2}$$

Think and Discuss

1. Explain why in a multiplication problem you need to write mixed numbers as improper fractions in order to multiply.

2. Explain using Problem 1, why $2 \cdot \frac{3}{8}$ is equal to $\frac{3}{8} + \frac{3}{8}$.

3. How do you write any whole number as a fraction?

LESSON **3-11** **Practice A**
Dividing Fractions and Mixed Numbers

Divide. Write each answer in simplest form.

1. $5 \div \frac{1}{2}$

2. $9 \div \frac{1}{3}$

3. $6 \div \frac{1}{4}$

4. $3 \div \frac{3}{4}$

5. $10 \div \frac{5}{6}$

6. $6 \div \frac{3}{8}$

Divide. Find each quotient in the box.

$\frac{1}{5}$	$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{6}{11}$	$\frac{5}{7}$	$\frac{7}{8}$	1	$1\frac{1}{2}$	2	$2\frac{6}{7}$	3	4	$5\frac{1}{3}$	$7\frac{1}{2}$
---------------	----------------	---------------	-----------------	---------------	---------------	---	----------------	---	----------------	---	---	----------------	----------------

7. $\frac{9}{5} \div \frac{3}{5}$

8. $\frac{6}{7} \div \frac{3}{7}$

9. $\frac{1}{6} \div \frac{5}{6}$

10. $\frac{1}{3} \div \frac{2}{3}$

11. $\frac{3}{4} \div \frac{1}{2}$

12. $\frac{1}{6} \div -\frac{2}{3}$

13. $2\frac{2}{3} \div \frac{1}{2}$

14. $1\frac{1}{4} \div \frac{1}{6}$

15. $2\frac{1}{2} \div \frac{7}{8}$

16. $2\frac{1}{2} \div 3\frac{1}{2}$

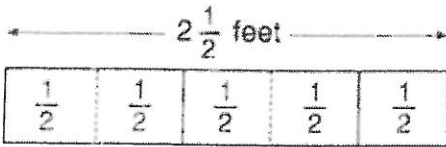
17. $1\frac{1}{6} \div 1\frac{1}{3}$

18. $-1\frac{1}{5} \div 2\frac{1}{5}$

19. A restaurant sells 3 sizes of soup. The medium is 8 ounces more than the small, and the large is twice as much as the medium. The large soup is 40 ounces. How many ounces is the small soup?
- _____

LESSON
3-11 **Reading Strategies**
Use a Visual Model

The Smith family has a two-and-a-half-foot-long sandwich to share. One-half foot of the sandwich will serve one person. How many one-half foot servings are in this sandwich?



Use the model to answer each question.

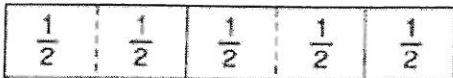
1. How long is the sandwich?

2. How long is each serving?

3. If you divided the sandwich into $\frac{1}{2}$ ft servings, how many would you have?

4. What is $2\frac{1}{2} \div \frac{1}{2}$?

Suppose you have two sandwiches.



5. How many feet are in both sandwiches?

6. What is $2\frac{1}{2} \times 2$?

7. Compare the answers to $2\frac{1}{2} \div \frac{1}{2}$ and $2\frac{1}{2} \times 2$. What do you notice?

LESSON
3-10
Review for Mastery
Dividing Fractions and Mixed Numbers

Dividing fractions and mixed numbers is very much like multiplying fractions and mixed numbers. Just follow these steps:

Step 1: Write any mixed numbers as improper fractions.

Step 2: Invert the divisor.

Step 3: Multiply and write the quotient in simplest form.

Divide: $1\frac{1}{8} \div \frac{1}{3}$

Step 1: $1\frac{1}{8} \div \frac{1}{3} = \frac{9}{8} \div \frac{1}{3}$

Step 2: $\frac{9}{8} \div \frac{1}{3} = \frac{9}{8} \cdot \frac{3}{1}$

Step 3: $\frac{9}{8} \cdot \frac{3}{1} = \frac{27}{8} = 3\frac{3}{8}$

Divide: $1\frac{1}{4} \div 3\frac{1}{3}$

Step 1: $1\frac{1}{4} \div 3\frac{1}{3} = \frac{5}{4} \div \frac{10}{3}$

Step 2: $\frac{5}{4} \div \frac{10}{3} = \frac{5}{4} \cdot \frac{3}{10}$

Step 3: $\frac{5}{4} \cdot \frac{3}{10} = \frac{15}{40} = \frac{3}{8}$

Divide. Write each answer in simplest form.

1. $\frac{4}{5} \div \frac{1}{2} = \frac{4}{5} \cdot \underline{\quad} = \underline{\quad} = \underline{\quad}$

2. $\frac{5}{8} \div \frac{5}{6} = \frac{5}{8} \cdot \underline{\quad} = \underline{\quad} = \underline{\quad}$

3. $2\frac{1}{2} \div 1\frac{3}{4} = \frac{5}{2} \div \frac{7}{4} = \frac{5}{2} \cdot \underline{\quad}$

4. $2\frac{2}{3} \div 1\frac{1}{5} = \frac{8}{3} \div \frac{6}{5} = \frac{8}{3} \cdot \underline{\quad}$

$= \underline{\quad} = \underline{\quad} = \underline{\quad}$

$= \underline{\quad} = \underline{\quad} = \underline{\quad}$

5. $\frac{3}{5} \div \frac{3}{10}$

6. $\frac{7}{8} \div \frac{1}{3}$

7. $\frac{5}{12} \div \frac{1}{2}$

8. $4\frac{1}{3} \div 1\frac{1}{9}$

9. $2\frac{1}{3} \div 1\frac{3}{4}$

10. $5\frac{5}{8} \div 2\frac{1}{2}$

LESSON
3-11 **Student Worksheet**
Dividing Fractions and Mixed Numbers

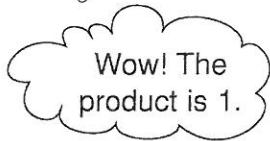
Problem 1

What is the reciprocal of $\frac{6}{7}$?

$$\frac{6}{7} \text{ FLIP } \frac{7}{6}$$



$$\frac{6}{7} \cdot \frac{7}{6} = \frac{\cancel{6}}{\cancel{7}} \cdot \frac{\cancel{7}}{\cancel{6}} = 1$$



Problem 2

Divide 9 by $1\frac{1}{2}$.

$$9 \div 1\frac{1}{2}$$

$$9 \div \frac{3}{2}$$

$$9 \cdot \frac{2}{3}$$

$$\frac{9}{1} \cdot \frac{2}{3} = \frac{18}{3} = \frac{6}{1} = 6$$

$$\frac{3}{2} \cdot \frac{2}{3} = 1$$

Think and Discuss

1. How is dividing fractions DIFFERENT from multiplying fractions?

2. What is the first step in dividing by mixed numbers?

LESSON **Practice A**
1-3 **Properties of Numbers**

Tell which property is shown.

1. $5 + 0 = 5$

2. $8 \cdot (6 \cdot 2) = (8 \cdot 6) \cdot 2$

3. $9 + 8 = 8 + 9$

4. $4 \cdot 1 = 4$

Simplify each expression. Write a reason for each step.

5. $13 + 28 + 7$

$13 + 28 + 7 = 28 + 13 + 7$

Reason: Commutative Property

$= 28 + (13 + 7)$

Reason: _____

$= 28 + \underline{\hspace{2cm}}$

Reason: Add.

$= \underline{\hspace{2cm}}$

Reason: _____

6. $20 \cdot (17 \cdot 5)$

$20 \cdot (17 \cdot 5) = 20 \cdot (\underline{\hspace{2cm}} \cdot 17)$

Reason: _____

$= (20 \cdot \underline{\hspace{2cm}}) \cdot 17$

Reason: _____

$= \underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}}$

Reason: Multiply.

$= \underline{\hspace{2cm}}$

Reason: _____

Use the Distributive Property to find each product.

7. $4(17)$

$4(17) = 4 \cdot (10 + \underline{\hspace{2cm}})$

$= (4 \cdot \underline{\hspace{2cm}}) + (4 \cdot 7)$

$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

8. $3(28)$

$3(28) = \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

LESSON
1-3

Reading Strategies

Use a Flowchart

Use a flowchart to help you simplify an expression, such as $(25 + 89) + 15$.

Step 1: Choose two numbers that are easy to add.

$$(25 + 89) + 15$$



Step 2: Rewrite the expression so the two numbers are next to each other.

Use the Commutative Property. $(25 + 89) + 15 = (89 + 25) + 15$



Step 3: Rewrite the expression so the two numbers are grouped together.

Use the Associative Property. $(89 + 25) + 15 = 89 + (25 + 15)$



Step 4: Add.

$$89 + (25 + 15) = 89 + 40 = 129$$

Use the expression $16 + (39 + 14)$ for Exercises 1–4.

1. Which two numbers are easy to add? _____
2. Rewrite the expression so that the numbers that are easy to add are next to each other. What property lets you do this?

3. Rewrite the expression so that the numbers that are easy to add are grouped together. What property lets you do this?

4. Simplify the expression. _____

Use the expression $35 + 47 + 5$ for Exercises 5–8.

5. Which two numbers are easy to add? _____

6. Rewrite the expression so that the numbers that are easy to add are next to each other. What property lets you do this?

7. Rewrite the expression so that the numbers that are easy to add are grouped together. What property lets you do this?

8. Simplify the expression. _____

LESSON **1-3** **Review for Mastery**
Properties of Numbers

You can use the Commutative Property, the Associative Property, and the Distributive Property with mental math to simplify expressions.

$16 + 47 + 14 = 47 + 16 + 14$	Commutative Property	$8 \cdot 3 \cdot 5 = 3 \cdot 8 \cdot 5$
$= 47 + (16 + 14)$	Associative Property	$= 3 \cdot (8 \cdot 5)$
$= 47 + 30$	Mental math	$= 3 \cdot 40$
$= 77$	Mental math	$= 120$
$9(28) = 9(20 + 8)$		$9(28) = 9(30 - 2)$
$= (9 \cdot 20) + (9 \cdot 8)$	Distributive Property	$= (9 \cdot 30) - (9 \cdot 2)$
$= 180 + 72$	Mental math	$= 270 - 18$
$= 252$	Mental math	$= 252$

Simplify each expression. Tell what properties you used.

1. $(45 + 39) + 25 = (39 + \underline{\hspace{1cm}}) + 25$ _____ Property
 $= 39 + (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$ _____ Property
 $= 39 + \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

2. $25 \cdot 7 \cdot 4 = 25 \cdot \underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}}$ _____ Property
 $= (\underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}}) \cdot \underline{\hspace{1cm}}$ _____ Property
 $= \underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

3. $5(18) = 5 \cdot (10 + \underline{\hspace{1cm}})$
 $= (5 \cdot \underline{\hspace{1cm}}) + (5 \cdot \underline{\hspace{1cm}})$
 $= \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$
 _____ Property

4. $6(29) = 6 \cdot (30 - \underline{\hspace{1cm}})$
 $= (6 \cdot \underline{\hspace{1cm}}) - (6 \cdot \underline{\hspace{1cm}})$
 $= \underline{\hspace{1cm}} - \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$
 _____ Property

LESSON
1-3 **Student Worksheet**
Properties of Numbers

Problem 1

Properties	
Commutative	
Add any order	$3 + 8 = 8 + 3$
Multiply any order	$5 \cdot 7 = 7 \cdot 5$
Associative	
Add any group	$(4 + 5) + 1 = 4 + (5 + 1)$
Multiply any group	$(9 \cdot 2) \cdot 6 = 9 \cdot (2 \cdot 6)$
Identity	
Add 0 , sum is number	$4 + 0 = 4$
Multiply by 1 , product is number	$8 \cdot 1 = 8$

Problem 2

Use Distributive Property to find $7(29)$	
Method 1: $7(29)$	$= 7 \cdot (20 + 9)$ Regroup $= 7 \cdot 20 = 140$ Multiply $= 7 \cdot 9 = 63$ $= 140 + 63$ Add So, $7(29) = 203$
Method 2: $7(29)$	$= 7 \cdot (30 - 1)$ Rewrite $= 7 \cdot 30 = 210$ Multiply $= 7 \cdot 1 = 7$ $= 210 - 7$ Subtract So, $7(29) = 203$

Which is easier for you, Method 1 or Method 2?

Think and Discuss

1. What is $25 \cdot 1$? Which property is represented?

2. Complete the expression $2 + (7 + 8) = (2 + 7) + \square$. How do you know this is the Associative Property?

3. Find $6 \cdot (9 + 14)$.

LESSON
1-8

Practice A

Solving Equations by Adding or Subtracting

Match each equation in Column A with its correct solution in Column B.

Column A	Column B	Column A	Column B
1. $n - 16 = 8$	A. $n = 12$	10. $x - 12 = 13$	L. $x = 14$
2. $5 = n - 7$	B. $n = 13$	11. $x + 8 = 40$	M. $x = 17$
3. $12 + n = 25$	C. $n = 17$	12. $34 = 16 + x$	N. $x = 18$
4. $n - 17 = 11$	D. $n = 24$	13. $x + 5 = 19$	P. $x = 25$
5. $n + 18 = 35$	E. $n = 27$	14. $4 + x = 52$	Q. $x = 32$
6. $7 = n - 28$	F. $n = 28$	15. $12 + x = 50$	R. $x = 33$
7. $n - 12 = 40$	G. $n = 35$	16. $15 = x - 2$	S. $x = 38$
8. $24 = n - 25$	H. $n = 49$	17. $52 = x + 9$	T. $x = 43$
9. $46 = n + 19$	J. $n = 52$	18. $x - 11 = 22$	U. $x = 48$

19. Chris has 55 baseball trading cards. He has 17 more cards than his sister Sara has. Write and solve an equation to find how many trading cards Sara has.

20. In 2008, Miguel Cabrera hit 37 home runs. His home run total was 11 fewer than the number of home runs that Ryan Howard hit the same year. Write and solve an equation to find how many home runs Ryan Howard hit in 2008.

LESSON
1-8

Reading Strategies

Follow a Procedure

In order to solve an equation, you must find a **solution**. A solution is a value of the variable that makes the equation true. To solve an equation, you need to get the variable by itself on one side of the equal sign.

- If you have an addition equation, you must subtract to get the variable by itself.
- If you have a subtraction equation, you must add to get the variable by itself.

Example:

$$z + 12 = 32 \quad \longleftarrow \quad \text{To get } z \text{ by itself, subtract } 12.$$

$$z + 12 - 12 = 32 - 12 \quad \longleftarrow \quad \text{Rewrite the equation to show that } 12 \text{ is subtracted from both sides.}$$

$$z = 20 \quad \longleftarrow \quad \text{This is the solution after subtracting } 12 \text{ from both sides.}$$

Check by using 12 in place of z .

$$20 + 12 \stackrel{?}{=} 32$$

$32 = 32$, so $z = 20$ is the correct solution.

Example:

$$27 = x - 8 \quad \longleftarrow \quad \text{To get } x \text{ by itself, add } 8.$$

$$27 + 8 = x - 8 + 8 \quad \longleftarrow \quad \text{Rewrite the equation to show that } 8 \text{ is added to both sides.}$$

$$35 = x \quad \longleftarrow \quad \text{This is the solution after adding } 8 \text{ to both sides.}$$

Check by using 35 in place of x .

$$27 \stackrel{?}{=} 35 - 8$$

$27 = 27$, so $x = 35$ is the correct solution.

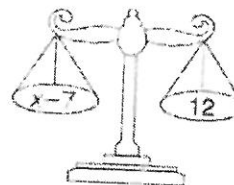
Use $m + 17 = 43$ for Exercises 1–4.

1. What operation is shown in this equation? _____
2. What operation will you use to get m by itself? _____
3. Rewrite the equation showing subtracting from both sides of the equation. _____
4. What is the value of m ? _____

LESSON **1-8** **Review for Mastery**

Solving Equations by Adding or Subtracting

Solving an equation is like balancing a scale. If you add the same weight to both sides of a balanced scale, the scale will remain balanced. You can use this same idea to solve an equation.



Think of the equation $x - 7 = 12$ as a balanced scale. The equal sign keeps the balance.

$-7 + 7 = 0$	$x - 7 = 12$	
	$x - 7 + 7 = 12 + 7$	Add 7 to both sides.
	$x + 0 = 19$	Combine like terms.
	$x = 19$	

When you solve an equation, the idea is to get the variable by itself. What you do to one side of the equation, you must do to the other side.

- To solve a subtraction equation, use addition.
- To solve an addition equation, use subtraction.

Solve and check: $y + 8 = 14$.

$+8 - 8 = 0$	$y + 8 = 14$	
	$y + 8 - 8 = 14 - 8$	Subtract 8 from both sides.
	$y + 0 = 6$	Combine like terms.
	$y = 6$	

Check: $y + 8 = 14$ To check, substitute 6 for y .

$$6 + 8 = 14$$

$$14 = 14$$

A true sentence, $14 = 14$, means the solution is correct.

Solve and check.

1. $x - 2 = 8$

$$x - 2 + \underline{\hspace{2cm}} = 8 + \underline{\hspace{2cm}}$$

$$x - 0 = \underline{\hspace{2cm}}$$

2. $b + 5 = 11$

$$b + 5 - \underline{\hspace{2cm}} = 11 - \underline{\hspace{2cm}}$$

$$b + 0 = \underline{\hspace{2cm}}$$

3. $n + 8 = 11$

4. $y - 6 = 2$

5. $a - 9 = 4$

6. $m + 2 = 18$

LESSON
1-8 **Student Worksheet**
Solving Equations by Adding or Subtracting

Problem 1

$$\begin{array}{r} x - 8 = 17 \\ + 8 \quad + 8 \\ \hline x = 25 \end{array}$$

Equations must stay balanced—with both sides equal.

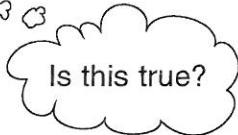
If a number is added to one side of an equation, the same number **must** be added to the other side.

Check:

$$\begin{array}{r} x - 8 = 17 \\ 25 - 8 \stackrel{?}{=} 17 \end{array}$$

Substitute $x = 25$.

Subtraction Property of Equality



Problem 2

$$\begin{array}{r} a + 5 = 11 \\ - 5 \quad - 5 \\ \hline a = 6 \end{array}$$

Equations must stay balanced—with both sides equal.

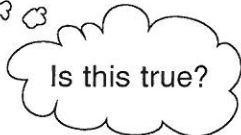
If a number is subtracted from one side of an equation, the same number **must** be subtracted from the other side.

Check:

$$\begin{array}{r} a + 5 = 11 \\ 6 + 5 \stackrel{?}{=} 11 \end{array}$$

Substitute $a = 6$.

Addition Property of Equality



Think and Discuss

1. Is $x = 25$ a solution to $x - 8 = 17$? Explain.

2. Why is 5 subtracted from both sides of the equation in Problem 2? What property is used?

3. How do you know that $a = 6$ is a solution to $a + 5 = 11$?

LESSON
1-9

Practice A
Solving Equations by Multiplying or Dividing

Solve.

1. $16 = n \div 2$

2. $\frac{e}{10} = 8$

3. $25 = \frac{x}{6}$

4. $18 = \frac{d}{3}$

5. $a \div 12 = 7$

6. $30 = b \div 4$

Solve and check.

7. $7w = 49$

8. $75 = 3x$

9. $60 = 12p$

10. $77 = 11m$

11. $4h = 48$

12. $9y = 54$

13. $2x = 30$

14. $45 = 5s$

15. $6z = 42$

16. The Fruit Stand charges \$0.50 each for navel oranges. Kareem paid \$4.00 for a large bag of navel oranges. How many oranges did he buy?

17. Jenny can type at a speed of 80 words per minute. It took her 20 minutes to type a report. How many words was the report?

18. At the local gas station, regular unleaded gasoline is priced at \$2.50 per gallon. If it cost \$37.50 to fill a car's gas tank, how many gallons of gasoline were purchased?

LESSON

1-9

Reading Strategies

Follow a Procedure

The opposite of multiplication is division: $\longrightarrow 12 \cdot 3 = 36$, and $36 \div 3 = 12$

The opposite of division is multiplication: $\longrightarrow 48 \div 12 = 4$, and $4 \cdot 12 = 48$

From these examples you can see that:

division “undoes” multiplication, and multiplication “undoes” division.

To solve multiplication and division equations:

- Get the variable by itself on one side of the equation.
- Keep the equation in balance by using the same operation on both sides.

Example:

$84 = 7x$ \longleftarrow Get the variable by itself. This is a multiplication equation, so divide to “undo” the multiplication.

$\frac{84}{7} = \frac{7x}{7}$ \longleftarrow Rewrite the equation to show that both sides are divided by 7.

$12 = x$ \longleftarrow This is the solution after dividing both sides by 7.

Check using 12 in place of x :

$$84 \stackrel{?}{=} 7(12)$$

$84 = 84$, so $x = 12$ is the solution.

Example:

$\frac{m}{15} = 8$ \longleftarrow Get the variable by itself. Multiply to “undo” division.

$\frac{m}{15} \cdot 15 = 8 \cdot 15$ \longleftarrow Rewrite the equation to show that both sides are multiplied by 15.

$m = 120$ \longleftarrow This is the solution after multiplying both sides by 15.

Check by using 120 in place of m .

$$\frac{120}{15} \stackrel{?}{=} 8$$

$8 = 8$, so $m = 120$ is the solution.

Use $108 = 9y$ for Exercises 1–3.

1. What operation will you use to solve the equation?

2. Rewrite the equation using the inverse operation on both sides.

3. What is the value of y ?

LESSON
1-9
Review for Mastery
Solving Equations by Multiplying or Dividing

When you solve an equation, you must get the variable by itself. Remember, what you do to one side of an equation, you must do to the other side.

- To solve a division equation, multiply both sides of the equation by the same number.

Solve and check: $\frac{a}{3} = 4$.

$$\frac{3a}{3} = 1a = a$$

$$\frac{a}{3} = 4$$

$$(3)\frac{a}{3} = 4(3)$$

$$a = 12$$

Multiply to solve a division equation.

Replace the variable with the solution.

Check: $\frac{a}{3} = 4$

$$\frac{12}{3} \stackrel{?}{=} 4$$

$$4 \stackrel{?}{=} 4 \quad \checkmark$$

A true sentence means the solution is correct.

Solve and check.

1. $\frac{x}{6} = 3$

2. $\frac{s}{8} = 8$

3. $\frac{c}{10} = 7$

4. $\frac{n}{3} = 12$

- To solve a multiplication equation, divide both sides of the equation by the same number.

Solve and check: $5k = 30$.

$$\frac{5k}{5} = 1k = k$$

$$5k = 30$$

$$\frac{5k}{5} = \frac{30}{5}$$

$$k = 6$$

Divide to solve a multiplication equation.

Replace the variable with the solution.

Check: $5k = 30$

$$5(6) \stackrel{?}{=} 30$$

$$30 \stackrel{?}{=} 30 \quad \checkmark$$

True

Solve and check.

5. $2w = 16$

6. $4b = 24$

7. $9z = 45$

8. $10m = 40$

LESSON
1-9 **Student Worksheet**
Solving Equations by Multiplying or Dividing

Problem 1

$$\frac{x}{7} = 20$$

$$\frac{7}{1} \cdot \frac{x}{7} = 20 \cdot 7$$

$$x = 140$$

x is being divided by 7.
 To isolate x, use the inverse operation: **MULTIPLY BOTH SIDES BY 7.**

Check:

Does $\frac{x}{7} = 20$ when $x = 140$?

$$\frac{140}{7} \stackrel{?}{=} 20 \quad \text{Substitute } x = 140.$$

$$20 = 20 \checkmark$$

Problem 2

$$240 = 4z$$

$$\frac{240}{4} = \frac{4z}{4}$$

$$60 = z$$

z is multiplied by 4.
 To isolate z, use the inverse operation: **DIVIDE BOTH SIDES BY 4.**

Think and Discuss

1. Why is 7 multiplied to both sides in Problem 1?

2. Is $z = 60$ a solution to the equation $240 = 4z$? Explain.

LESSON **Practice A**
2-5 *Solving Equations Containing Integers*

Solve each equation. Check your answer.

1. $n - 6 = -2$

2. $x - 8 = -11$

3. $7 = a - 5$

4. $y + 4 = 2$

5. $c + 7 = -3$

6. $0 = v + 1$

7. $8j = -16$

8. $-3k = 24$

9. $-20 = -4s$

10. $\frac{m}{-2} = -5$

11. $\frac{d}{6} = -3$

12. $\frac{r}{-7} = 4$

13. $p + 8 = -6$

14. $-15 = 5b$

15. $f - 9 = -1$

16. $\frac{n}{6} = -4$

17. $k + 10 = 3$

18. $4a = -16$

19. $-6x = -36$

20. $2 = e - 7$

21. $3 = \frac{m}{2}$

22. The temperature in Minnesota was -8°F one day. This was 12 degrees less than the temperature in Indiana on the same day. What was the temperature in Indiana?

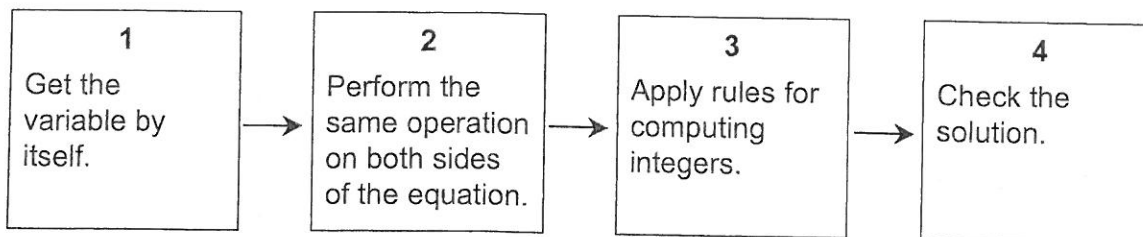
23. Mr. Harding sold 100 shares of stock at \$14 per share. He had a loss of \$6 per share. What did Mr. Harding pay for each share of the stock?

LESSON **Reading Strategies**

2-5 **Use a Flowchart**

The rules for solving equations with integers are the same as with whole numbers.

Use a flowchart to help you follow the rules.



$x + 12 = 5$	→ Get x by itself.
$x + 12 - 12 = 5 - 12$	→ Subtract 12 from both sides.
$x = 5 + (-12)$	→ Add the opposite.
$x = -7$	
$x + 12 = 5$	→ Check.
$-7 + 12 \stackrel{?}{=} 5$	
$5 = 5$	

Use $w - 12 = (-4)$ to answer Exercises 1–4.

1. What operation is used in this equation?

2. What operation will you use to get the variable by itself?

3. Apply this operation to both sides of the equation.

4. What is the value of w ?

Use $x + (-9) = (-4)$ to answer Exercises 5 and 6.

5. What operation is used in this problem?

6. What operation will you use to get x by itself?

LESSON
2-5

Review for Mastery

Solving Equations Containing Integers

- You can use addition to solve an equation involving subtraction. Addition undoes subtraction. Adding the same number to both sides of the equation keeps the equation balanced.

$x - 5 = -6$	Check
$x - 5 + 5 = -6 + 5$	$x - 5 = -6$
$x = -1$	$-1 - 5 \stackrel{?}{=} -6$
	$-6 \stackrel{?}{=} -6 \checkmark$

- You can use subtraction to solve an equation involving addition. Subtraction undoes addition. Subtracting the same number from both sides of the equation keeps the equation balanced.

$n + 4 = -15$	Check
$n + 4 - 4 = -15 - 4$	$n + 4 = -15$
$n = -19$	$-19 + 4 \stackrel{?}{=} -15$
	$-15 \stackrel{?}{=} -15 \checkmark$

Solve. Check your answer.

1. $p - 9 = -3$
 $p - 9 + \underline{\hspace{1cm}} = -3 + \underline{\hspace{1cm}}$

2. $w - 2 = -14$
 $w - 2 + \underline{\hspace{1cm}} = -14 + \underline{\hspace{1cm}}$

3. $x - 12 = -5$
 $x - 12 + \underline{\hspace{1cm}} = -5 + \underline{\hspace{1cm}}$

4. $f - 8 = 6$
 $f - 8 + \underline{\hspace{1cm}} = 6 + \underline{\hspace{1cm}}$

5. $6 = m - 7$

6. $-4 = s - 10$

7. $-8 = y - 2$

8. $a + 19 = 7$

9. $b + 15 = -9$

10. $39 + t = 45$

11. $-5 = x + 7$

12. $-2 = k + 11$

13. $10 = -3 + j$

LESSON
2-5

Review for Mastery
Solving Equations Containing Integers (continued)

- You can use division to solve an equation involving multiplication.
Division undoes multiplication. Dividing both sides of the equation by the same number keeps the equation balanced.

$$3y = -9$$

$$\frac{3y}{3} = \frac{-9}{3}$$

$$y = -3$$

Check

$$3y = -9$$

$$3 \cdot (-3) \stackrel{?}{=} -9$$

$$-9 \stackrel{?}{=} -9 \checkmark$$

- You can use multiplication to solve an equation involving division.
Multiplication undoes division. Multiplying both sides of an equation by the same number keeps the equation balanced.

$$\frac{a}{-5} = -8$$

$$-5 \cdot \frac{a}{-5} = -8 \cdot (-5)$$

$$a = 40$$

Check

$$\frac{a}{-5} = -8$$

$$\frac{40}{-5} \stackrel{?}{=} -8$$

$$-8 \stackrel{?}{=} -8 \checkmark$$

Solve. Check your answer.

14. $5g = -35$

$$\frac{5g}{5} = \frac{-35}{5}$$

15. $-8y = -96$

$$\frac{-8y}{-8} = \frac{-96}{-8}$$

16. $54 = -6f$

$$\frac{54}{-6} = \frac{-6f}{-6}$$

17. $3e = -33$

18. $-49 = 7n$

19. $-75 = -5c$

20. $\frac{n}{4} = -15$

21. $\frac{m}{-6} = -9$

22. $\frac{s}{-10} = 8$

23. $4 = \frac{w}{-6}$

24. $9 = \frac{z}{5}$

25. $-11 = \frac{h}{6}$

LESSON 2-5 Student Worksheet
Solving Equations Containing Integers

Problem 1

How do you solve this equation?

$$n + 3 = -10$$

$$\begin{array}{r} -3 \quad -3 \\ \hline \end{array}$$

Subtract 3 from both sides.

The solution to the equation is $n = -13$.

Problem 2

Sometimes you need to multiply both sides by a number in order to isolate the variable.

$$\frac{a}{-3} = 9$$

Hmm... do I multiply by 9 or -3 ?

Multiply by -3 to isolate the variable.

The solution to the equation is $a = -27$.

Think and Discuss

- In Problem 1, what is the variable? _____
- How do you "undo" the addition to isolate n in Problem 1?

- Explain what it means that $n = -13$ is a solution to $n + 3 = -10$.

Remember... A number plus its opposite is zero.

$6 + (-6) = 0$

- When is $n + 1$ equal to zero? _____
- When is $n - 1$ equal to zero? _____
- When is $-n + 1$ equal to zero? _____
- When is $-n - 1$ equal to zero? _____

LESSON
2-6

Practice A
Solving Two-Step Equations

Solve each equation. Cross out each number in the box that matches a solution.

- | | | | | | | | | | | |
|---|----|----|----|-----|----|---|---|---|---|----|
| 6 | -8 | -2 | -6 | -18 | -3 | 2 | 4 | 8 | 3 | 18 |
|---|----|----|----|-----|----|---|---|---|---|----|

1. $5x + 8 = 23$

2. $-2p - 4 = 2$

3. $6a - 11 = 13$

4. $4n + 12 = 4$

5. $9g + 2 = 20$

6. $\frac{k}{6} + 8 = 5$

7. $\frac{s}{3} - 4 = 2$

8. $\frac{c}{2} + 5 = 1$

9. $9 + \frac{a}{6} = 8$

Solve. Check each answer.

10. $3v - 12 = 15$

11. $8 + 5x = -2$

12. $\frac{d}{4} - 9 = -3$

13. An electrician charges \$50 to come to your house. He also charges \$25 for each hour he spends at your house. The electrician charges you a total of \$125. How many hours did he spend at your house?

LESSON **2-6** **Reading Strategies**
Follow a Procedure

To solve two-step equations, follow these steps.

To Solve Two-Step Equations

$3n + 5 = 23$

Step 1: Get the variable term by itself. Use the inverse operation.

$$3n + 5 - 5 = 23 - 5$$

$$3n = 18$$

Subtract 5 from both sides.

Step 2: Get the variable by itself. Use the inverse operation.

$$\frac{3n}{3} = \frac{18}{3}$$

Divide both sides by 3.

Step 3: Compute and simplify the solution.

$$n = 6$$

Answer the following questions.

1. What is the first step in solving a two-step equation?

2. Which term in the equation above does not contain a variable?

3. What operation was performed to remove that term?

4. What is the second step in solving a two-step equation?

5. Which term in the equation contains a variable?

6. What operation was performed to get the n by itself?

7. What is the third step in a two-step equation?

LESSON
2-6

Review for Mastery

Solving Two-Step Equations

You can solve two-step equations by undoing one operation at a time. First undo any addition or subtraction, then undo any multiplication or division.

Complete the steps to solve each equation.

1. $7x + 3 = 31$

$7x + 3 - \underline{\quad} = 31 - \underline{\quad}$ ← Subtract $\underline{\quad}$ from both sides to undo addition.

$7x = 28$

$\frac{7x}{7} = \frac{28}{7}$ ← Divide both sides by $\underline{\quad}$ to undo multiplication.

$x = 4$

Check

$7x + 3 = 31$

$7(\underline{\quad}) + 3 \stackrel{?}{=} 31$ ← Substitute $\underline{\quad}$ for x .

$\underline{\quad} + 3 \stackrel{?}{=} 31$

$31 \stackrel{?}{=} 31 \checkmark$ ← 4 is a solution.

2. $\frac{n}{6} - 8 = 4$

3. $8a - 5 = 11$

4. $9 + \frac{w}{2} = 12$

$\frac{n}{6} - 8 + \underline{\quad} = 4 + \underline{\quad}$

$8a - 5 + \underline{\quad} = 11 + \underline{\quad}$

$9 - \underline{\quad} + \frac{w}{2} = 12 - \underline{\quad}$

$\frac{n}{6} = 12$

$8a = \underline{\quad}$

$\frac{w}{2} = \underline{\quad}$

$6 \cdot \frac{n}{6} = \underline{\quad} \cdot 12$

$\frac{8a}{8} = \frac{16}{8}$

$2 \cdot \frac{w}{2} = \underline{\quad} \cdot 3$

$n = \underline{\quad}$

$a = \underline{\quad}$

$w = \underline{\quad}$

Solve.

5. $4n + 11 = 27$

6. $\frac{z}{7} - 6 = 3$

7. $3 - 2k = -7$
